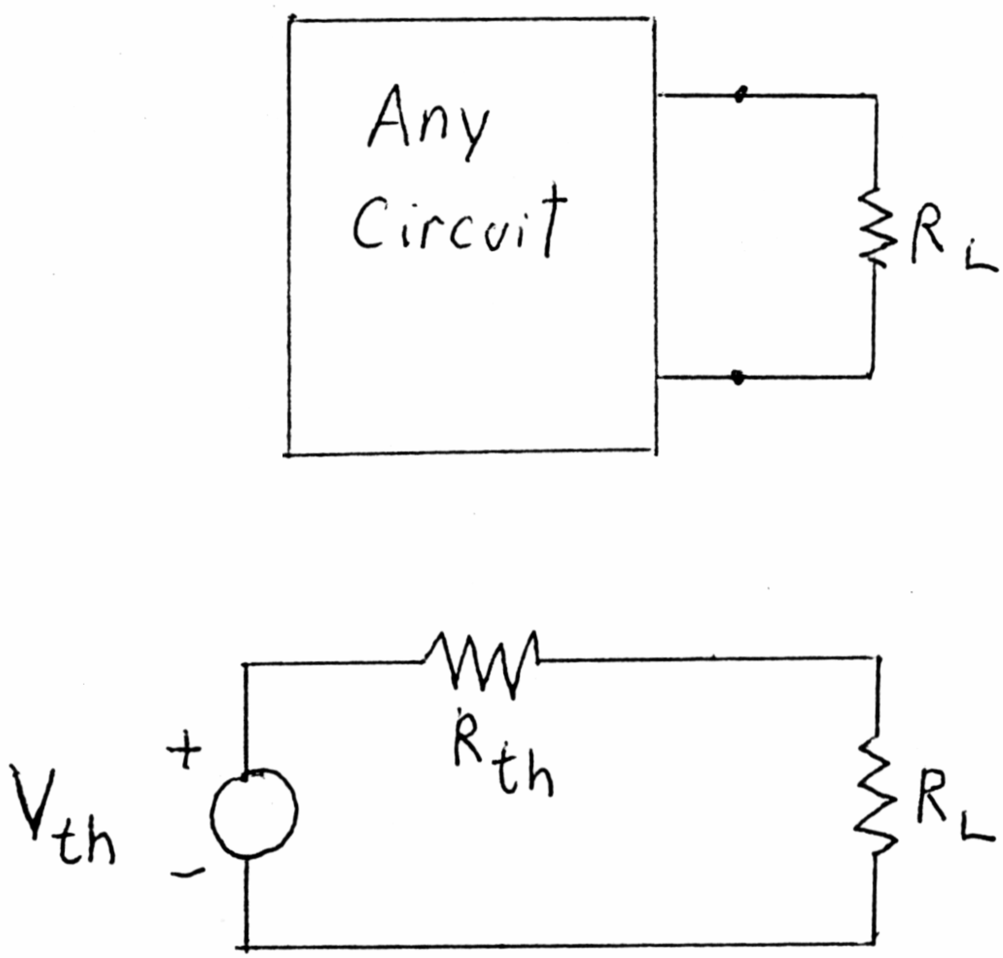
Thevenin Equivalent Circuits (EC 4.10)

Thevenin equivalent

- Current delivered to any load resistance by a circuit is equal to:
- Voltage source equal to open circuit voltage V_{th} at load
- In series with a simple resistor R_{th} (the source impedance).



Procedure for Finding Thevenin Equivalent

(1) Remove all elements not included in the circuit,

- Remove all loads at the output.

(2) Find the open circuit voltage = V_{th}

(3) Do one of the following to get R_{th}:

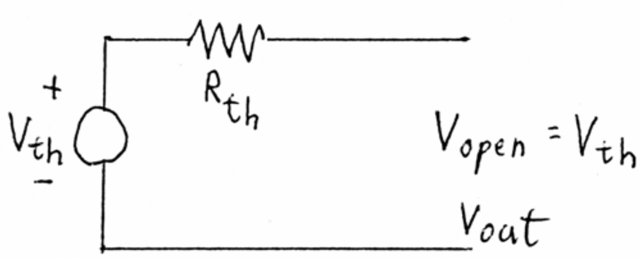
(3a) Find the output resistance R_{th}:

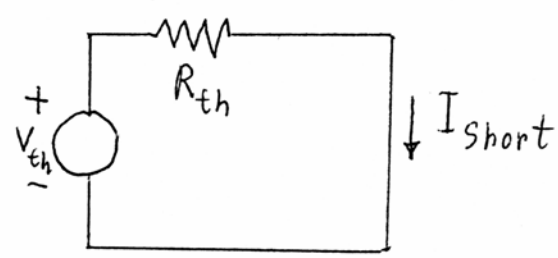
- Turn off all sources
- Voltage sources become shorts
- Current sources become open
- Then calculate resistance of circuit as seen from output.

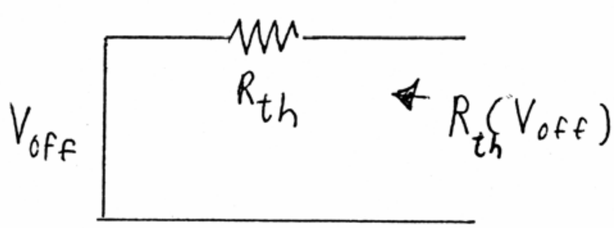
(3b) Find short circuit current:

- Calculate the load resistance by

$$R_{th} = \frac{V_{th}}{I_{short}}$$







Example Thevenin Equivalents

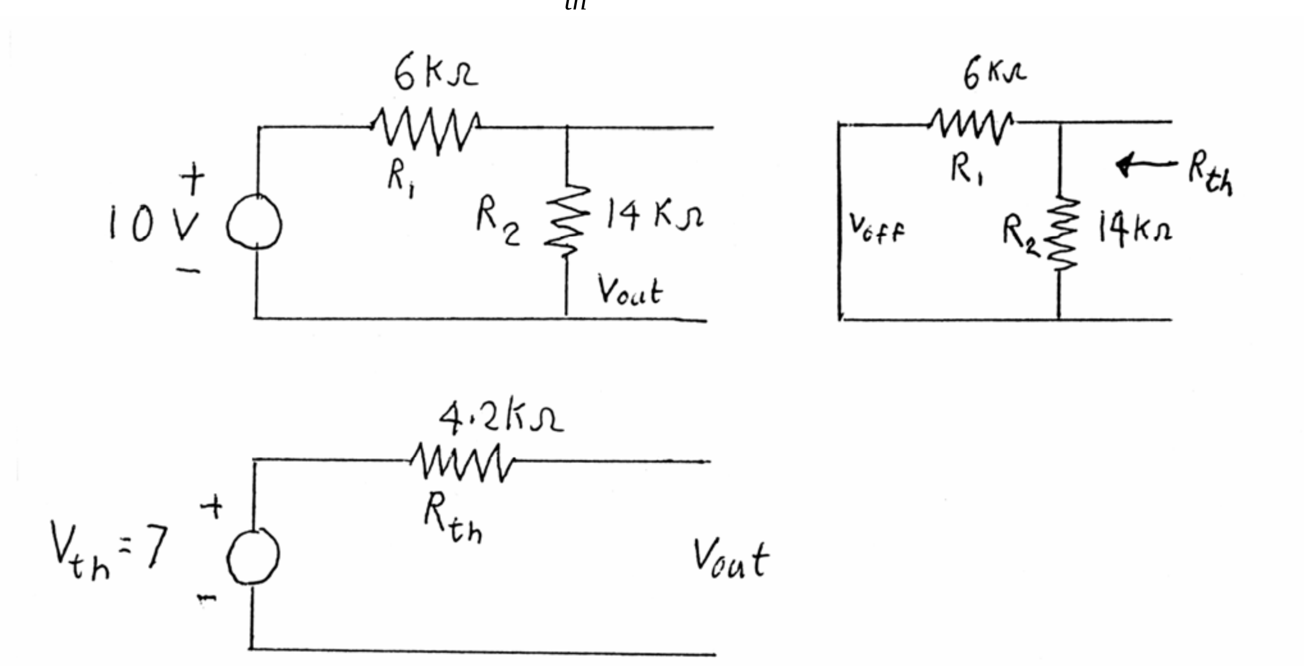
- Consider the simple voltage divider circuit
- Finding V_{th} by open circuit voltage
- This acts a simple voltage divider

$$V_{th} = V_{open} = \frac{R_2}{R_1 + R_2} V = \frac{14000}{6000 + 14000} 10 = 7\ V$$

- Setting sources off (short voltage source)
- Then input resistance is parallel resistors

$$\frac{1}{R_{th}} = \frac{1}{R_{tin}} = \frac{1}{14000} + \frac{1}{6000} = 2.38 \times 10^{-4}\ mhos$$

$$R_{th} = 4.2\ K\Omega$$



Example Thevenin Equivalents

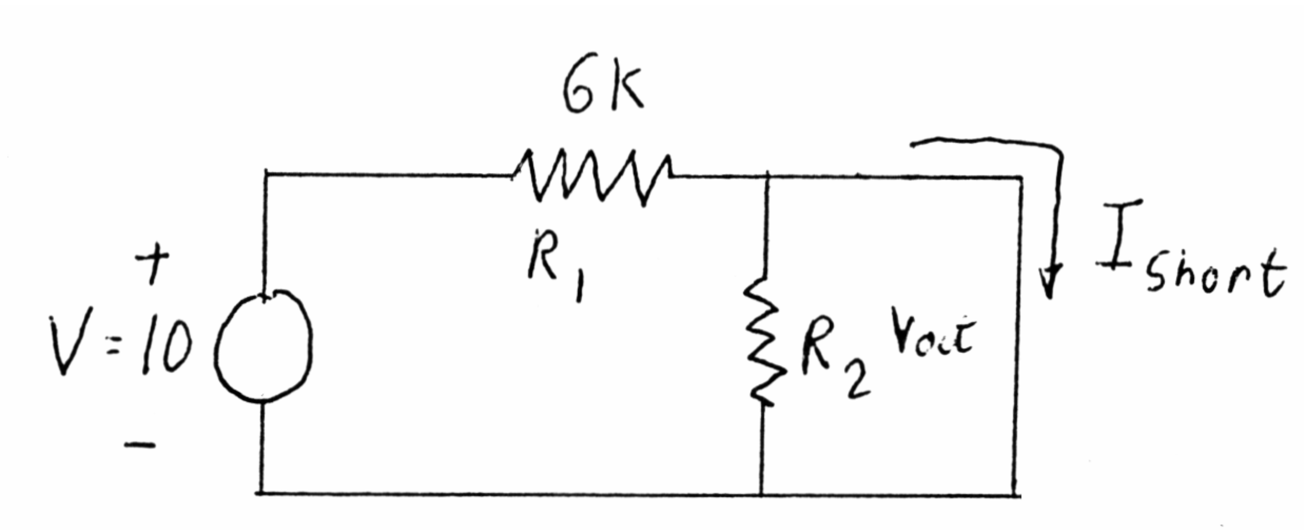
- Using alternate method for R_{th}
- Short the output
- Hence R_2 removed
- Short circuit current is

$$I_{short} = \frac{V}{R_1} = \frac{10}{6000} = 1.667\ mA$$

- Then

$$R_{th} = \frac{V_{th}}{I_{short}} = \frac{7}{0.00166} = 4.2\ K\Omega$$

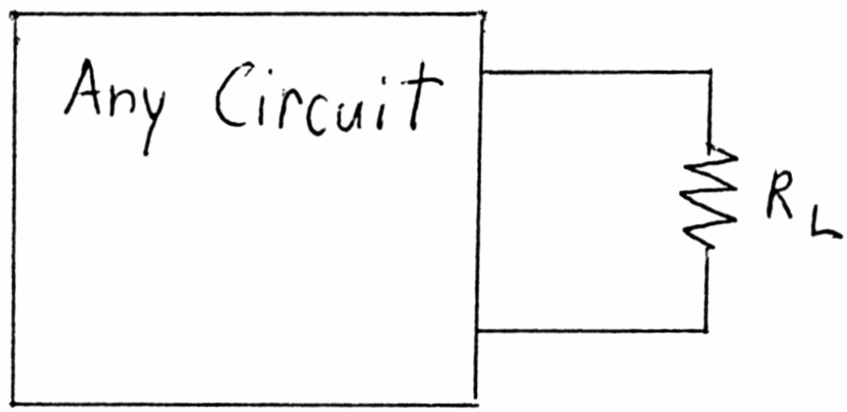
- Note often easier to use short output method.

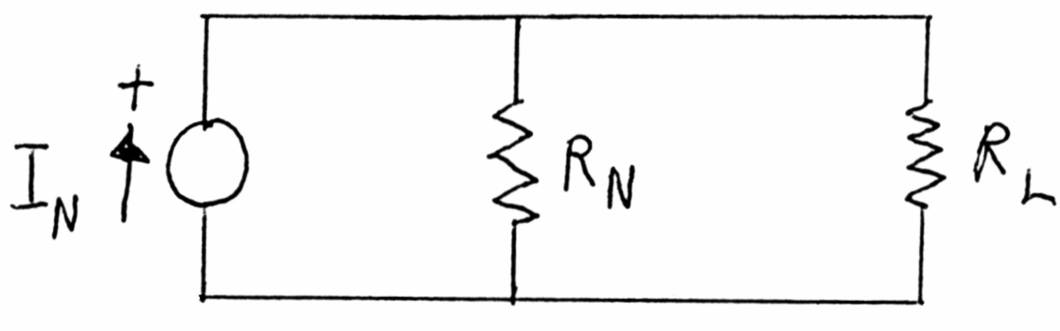


Norton Equivalent Circuits

Norton Equivalent

- Current delivered to any load resistance by a circuit is equal to:
- Constant current source = to short circuit current I_N at the load
- Shunt resistor R_N = resistance circuit when all sources are stopped.





Procedure for Norton Equivalent Circuits

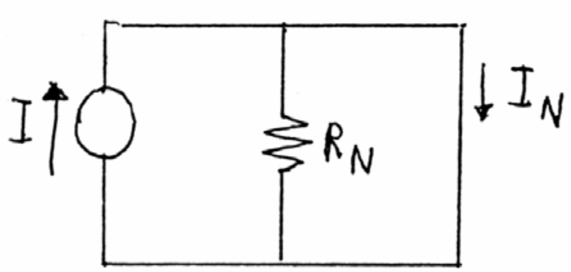
(1) Remove all elements not included in the circuit
- Remove all loads at the output.

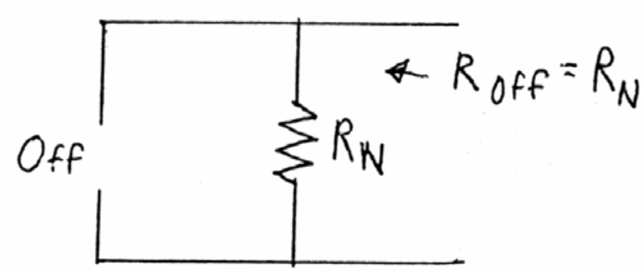
(2) Find the short circuit current = I_N

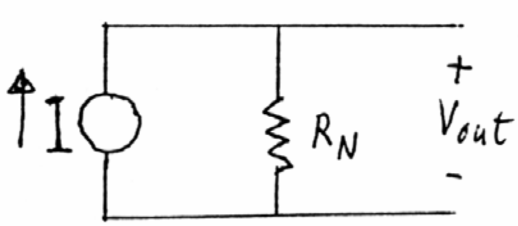
(3) Do one of the following:
(3a) Find the output resistance: turning off all constant sources
- Voltage sources become shorts
- Current sources become open
- Then calculate resistance of circuit as seen from output.

(3b) Find the open circuit voltage V_{open}
- Calculate the Norton resistance R_N by

$$R_N = \frac{V_{open}}{I_N}$$







Example Norton Equivalent Circuit

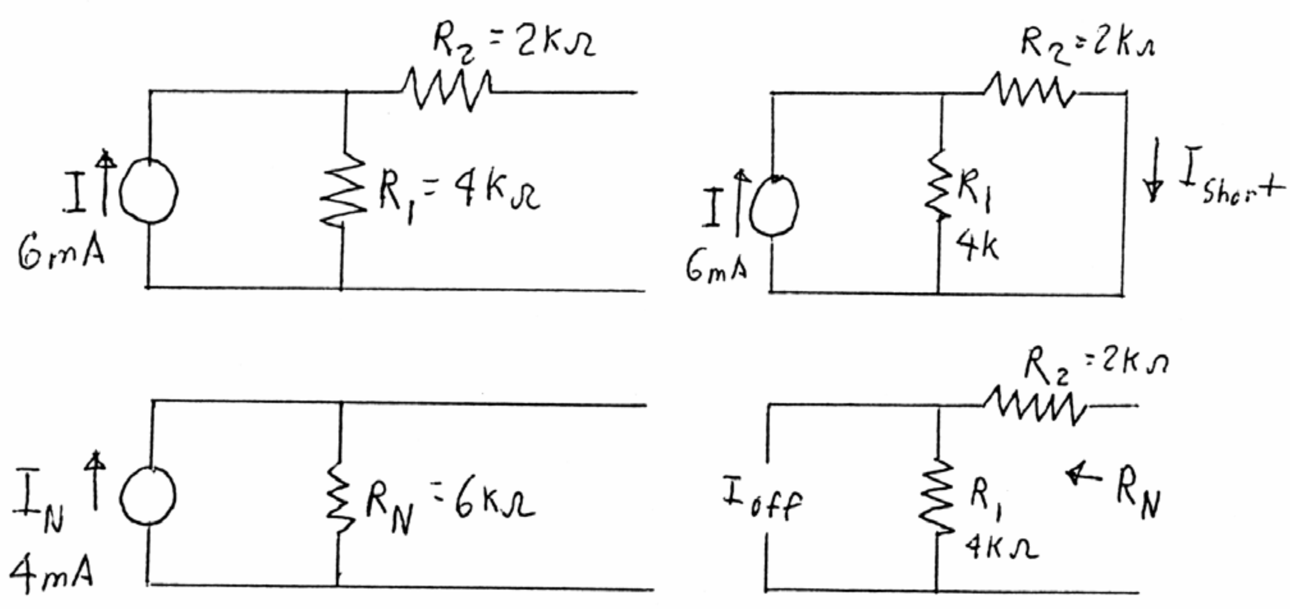
- Consider a current source with R_1 in parallel & R_2 on output
- Find the short circuit current = I_N
- When shorted becomes a simple current divider

$$I_N = I_{short} = I \frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

$$I_N = I \frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]} = 0.006 \frac{\left[\frac{1}{2000}\right]}{\left[\frac{1}{4000} + \frac{1}{2000}\right]} = 0.006 \frac{0.0005}{0.00075} = 4\ mA$$

- Then find the output resistance
- Setting the current source off (open)
- Now R_1 and R_2 in series

$$R_N = R_{out} = R_1 + R_2 = 2000 + 4000 = 6\ K\Omega$$



Example Norton Equivalent Circuit Con'd

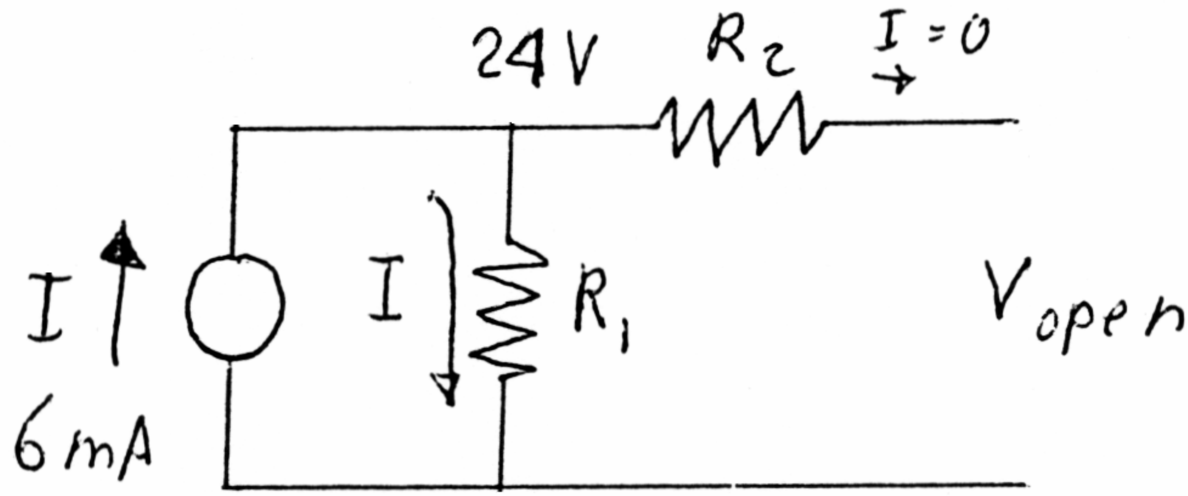
- Alternate way for R_N
- Finding the open circuit voltage

$$V_{open} = IR_2 = 0.006 \times 4000 = 24\,V$$

- Thus the output resistance is

$$R_N = \frac{V_{open}}{I_N} = \frac{24}{0.004} = 6\ K\Omega$$

- Depending on circuit this may be faster then shutting off source



Relationship Between Thevenin and Norton Circuits

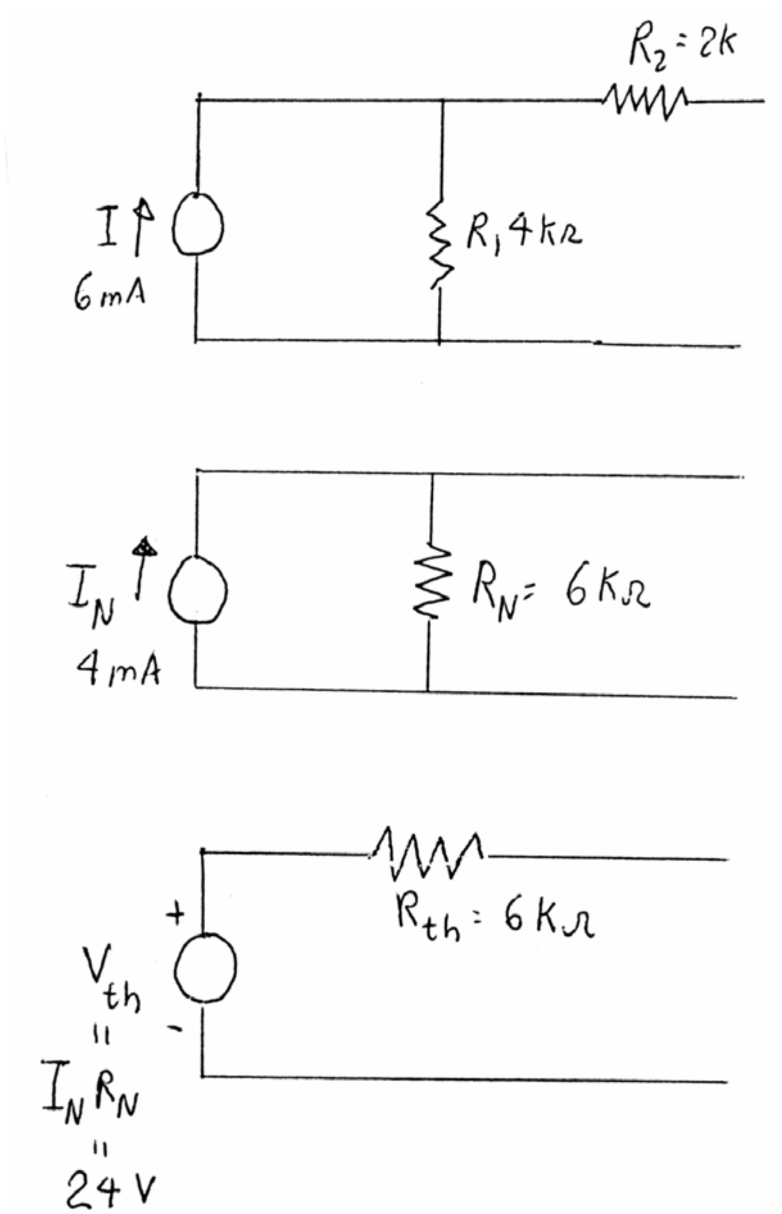
- Can easily change Thevenin into Norton or vice versa
- By definition

$$R_{th} = R_N$$

- Thus

$$V_{th} = I_N R_N = I_N R_{th}$$

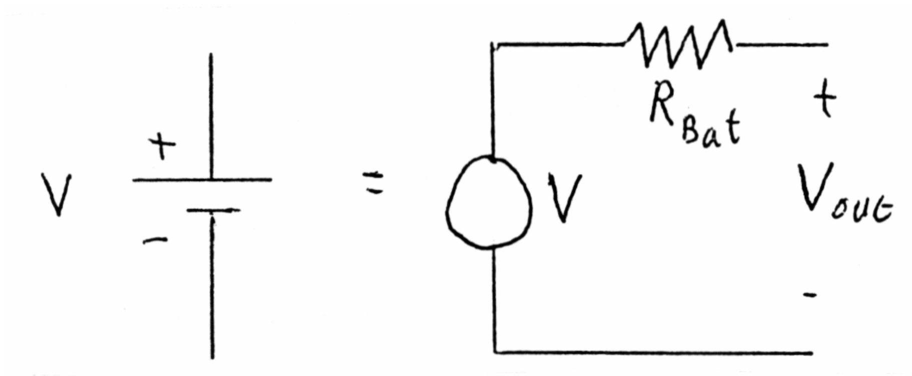
$$I_N = \frac{V_{th}}{R_{th}} = \frac{V_{th}}{R_N}$$



When to Use Thevenin and Norton Circuits

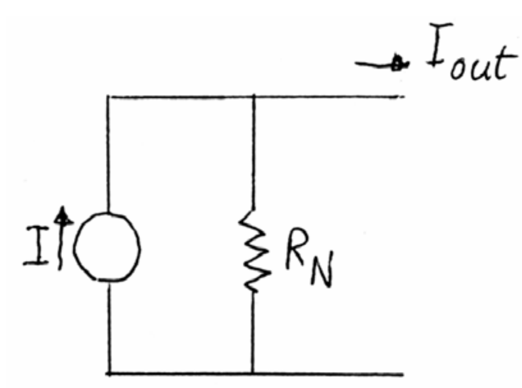
Thevenin equivalents most useful for:

- Where the information wanted is a single number
- Such as the current out of a given line.
- Circuits similar to non-ideal voltage sources
- Eg Battery or simple power supply (50 ohm input)
- There R_{th} provides current limit & internal resistance



Norton equivalents are most useful for:

- Circuits that approximate a non-ideal current source.
- Eg. current limited power supply
- There R_N creates the voltage limit & internal resistance



Superposition of Elements

- Another way of solving circuits with linear circuit elements
- With linear circuits the effect of the combined system is linear
- Obtained by adding together effect of each source on circuit

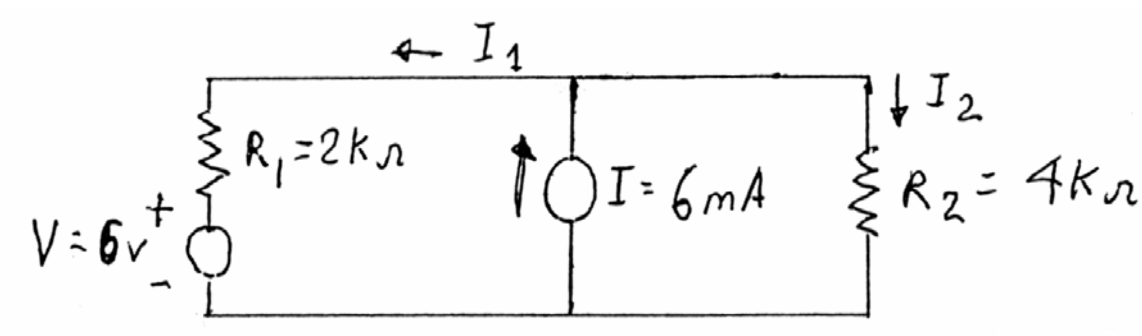
- To use superposition analysis:

(1) Set all power sources except one to zero
- Short circuit all voltage sources
- Open circuit all current sources
- These can be thought of as simplified circuits

(2) Calculate the voltages and currents from that one source

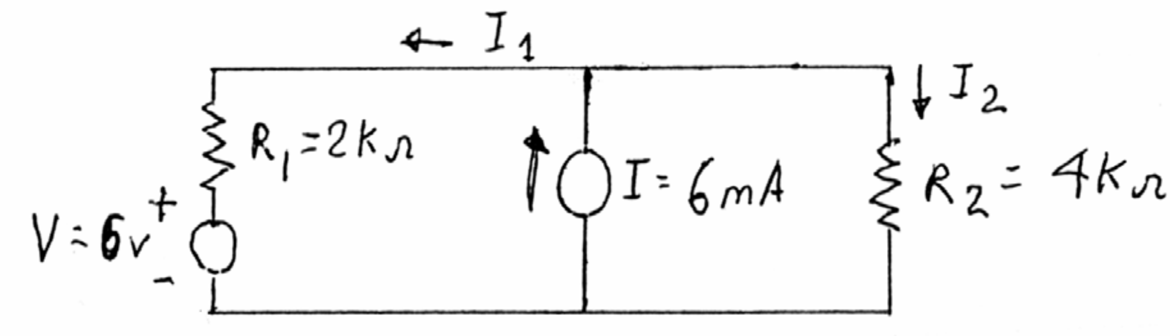
(3) Repeat 1-2 for each source individually

(4) Final result, at each point in the circuit:
- Sum the voltages and currents of all the simplified circuits
- Result is the combined voltage/current of the circuit.



Example of Superposition

- Solve the circuit below with a current and voltage source

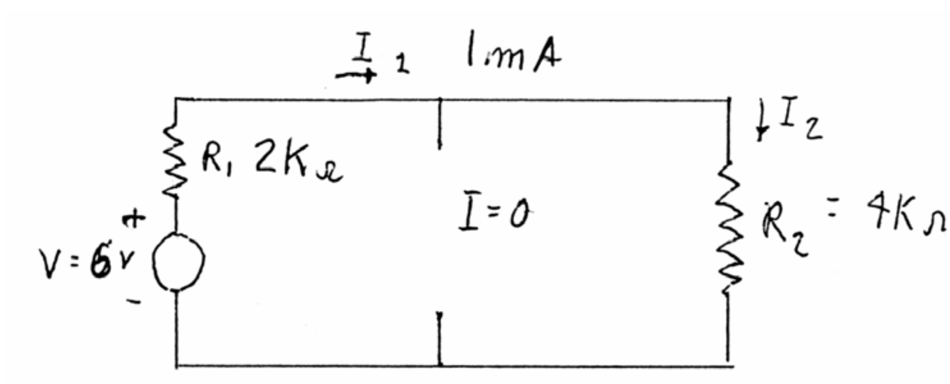


- First turn off (open circuit) the current source
- Then really only have two resistors in series:

$$I_1 = \frac{V}{R_1 + R_2} = \frac{6}{2000 + 4000} = 1\,mA$$

- Using a voltage divider for the R_2 line

$$V_{R1} = V\frac{R_2}{R_1 + R_2} = 6\frac{4000}{2000 + 4000} = 4\,V$$



Example of Superposition Continued

- Now turn on the current source
- Turn off the voltage source (short circuit it)
- Then using the current divider formula

$$I_2 = I \frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

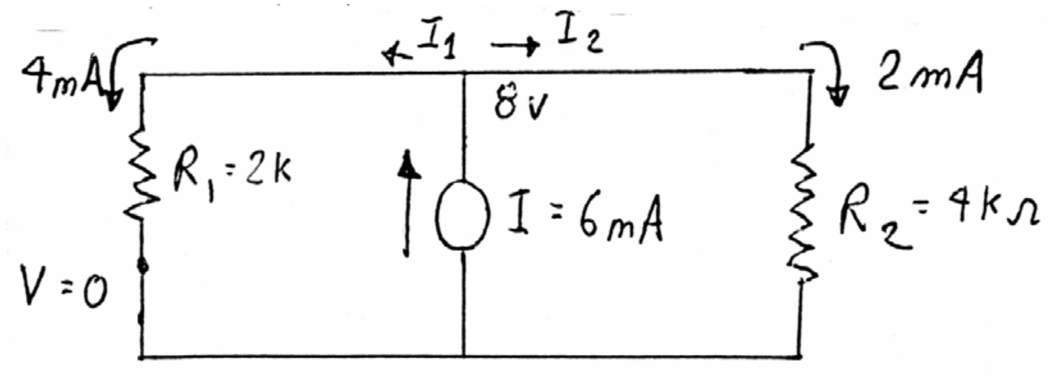
$$I_2 = I \frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]} = 0.006 \frac{\left[\frac{1}{4000}\right]}{\left[\frac{1}{4000} + \frac{1}{2000}\right]} = 0.006 \frac{0.00025}{0.00075} = 2\ mA$$

- Similarly for the R_1 branch

$$I_1 = I \frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]} = 0.006 \frac{\left[\frac{1}{2000}\right]}{\left[\frac{1}{4000} + \frac{1}{2000}\right]} = 0.006 \frac{0.0005}{0.00075} = 4\ mA$$

- The voltage across the R_2 is

$$V_{R2} = I_2 R_2 = 0.002 \times 4000 = 8\,V$$



Example of Superposition Continued

- Now add the voltages and currents
- For the voltage across R_2

$$V_{R2} = V_{R2I-on} + V_{R2V-on} = 8 + 4 = 12\,V$$

- For the current in R_2

$$I_{R2} = I_{R2I-on} + I_{R2V-on} = 2 + 1 = 3\ mA$$

- For the current in R_1
- Note: the V and I source only currents are in opposite directions

$$I_{R1} = I_{R1I-on} + I_{R1V-on} = 4 - 1 = 3\ mA$$

- Voltage drop across R_1

$$V_{R1} = I_{R1} R_1 = 0.003 \times 2000 = 6\ V$$

- With this we have a full analysis of the circuit